# Confidence Intervals and Hypothesis Tests: Two Samples

### 9.5  t-Interval, Two Population Means: Independent Samples (Unequal Variances)

1. Sensa is a product designed to help its users exercise portion control.  It uses something called tastant crystals.  A user sprinkles these tastant crystals onto their food prior to eating.  In an independent, double-blind, placebo-controlled study 29 subjects used Sensa for six months, and 27 subjects used a placebo for six months.  The dieters did not make any other changes to their lifestyle.  The Sensa group lost an average of 27.58 pounds with a standard deviation of 9.7 pounds while the placebo group lost 1.10 pounds with a standard deviation of 1.9 pounds.  Construct a 99% confidence interval to compare the average weight loss from Sensa and placebo over six months of use (assume unequal variances).

2. In May of 2010 the IMA (a professional organization for accountants and financial professionals) conducted a survey of its members living in Middle-Eastern countries.  Eleven respondents from Lebanon reported an average base salary of $30,829 with a standard deviation of $36,550.  Twelve respondents from Bahrain reported an average base salary of $33,400 and a standard deviation of $27,317.  Form a 95% confidence interval to compare the average base salary of IMA members in Lebanon and Bahrain (assume unequal variances).

3. The 2011 In-House Counsel Compensation Survey (a survey of in-house attorneys) found that there was a difference between the salaries of male and female attorneys working as in-house counsel.  Use the following data to form a 98% confidence interval to compare the salaries of male and female in-house counsels.  Is there a significant difference (assume unequal variances)?

| Males | | Females | |
|---|---|---|---|
| $n$ | 29 | $n$ | 29 |
| $\bar{x}$ | $218,423 | $\bar{x}$ | $179,450 |
| $s$ | $75,833 | $s$ | $64,167 |

Answers:

1. Since the interval is entirely positive, it seems very clear that Sensa produces a greater amount of weight loss than placebo. We are 99% confident that the true average difference in weight loss is between 21.43 pounds and 31.53 pounds.

$$d.f. = 30.29987144 = 30$$

$$t_{\alpha/2} = 2.75$$

$$E = 2.75 * \sqrt{\frac{9.7^2}{29} + \frac{1.9^2}{27}} \approx 5.054457$$

$$[26.48 - 5.05, 26.48 + 5.05] = [21.43, 31.53]$$

2. Based on the sample data and the resulting interval, we cannot conclude that a significant difference exists between the wages in Lebanon and Bahrain. Since zero is inside the interval, it is possible that there is no difference between the two countries wages for IMA professionals.

$$d.f. = 18.4621728 = 18$$

$$t_{\alpha/2} = 2.101$$

$$E = 2.101 * \sqrt{\frac{36550^2}{11} + \frac{27317^2}{12}} \approx 28,470.72503$$

$$[-\$31,041.73, \$25,899.73]$$

3. Since the interval contains zero, we cannot conclude that males are earning significantly more than females. However, the interval is very wide and very lopsided, so perhaps if we had used a 95% confidence level it would have shown a difference. Finally, note that we use the simpler formula to calculate the degrees of freedom here since the sample sizes are the same.

$$d.f. = 29 + 29 - 2 = 56$$

$$t_{\alpha/2} = 2.396$$

$$E = 2.396 * \sqrt{\frac{75833^2}{29} + \frac{64167^2}{29}} \approx 44,198.08335$$

$$[-\$5,225.1, \$83,171.1]$$